HEAT AND MASS TRANSFER DURING SUBLIMATION AND CONDENSATION IN A CONICAL ANNULUS

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A solution is given for thermostatic control of conical and spherical objects under conditions of one-sided radiative heating in a vacuum.

We have [1] considered a cylindrical heat-transmitting ring rotating around an axis as a means of efficient thermostatic control for an object subject to one-sided heating in a vacuum.

Considerable practical interest attaches also to conical and spherical heat-transmitting elements working under similar conditions (rotating ring screens).

Consider a thermostat constructed as two identical conical shells (Fig. 1a) having a common base (it is assumed that the base is ideally insulated in discussing one of these shells, in order to eliminate and effects). Each shell consists of two thin-walled components, the gap between which is free from noncondensable gases and filled with a certain amount of working body, whose triple point lies above the temperatures maintained in the thermostatic ring.

The thermostat rotates in a vacuum around its axis OO_1 and is exposed to a parallel beam of shortwave radiation perpendicular to that axis; the outer skin radiates in accordance with Stefan's law into outer space, so the resultant heat flux through the outer skin is

$$q=\begin{cases}-\varepsilon\sigma T_w^4 & \text{for}\quad \varphi<\dfrac{\pi}{2}\ \text{and}\ \varphi>\dfrac{3\pi}{2},\\[2mm] -A_sE\cos\beta\cos\varphi-\varepsilon\sigma T_w^4 & \text{for}\quad \dfrac{\pi}{2}<\varphi<\dfrac{3\pi}{2}.\end{cases}\tag{1}$$

The outer skin is coated on the inside with a layer of solid condensate; the rate of the phase transition in the absence of heat transfer at the inner surface is

$$J_m=\frac{q}{L}-\frac{\omega(c'\rho'\delta'+c_w\rho_w\delta_w)}{L}\cdot\frac{dT_m}{d\varphi}.\tag{2}$$

It is assumed that the heat flow along the jacket is negligible by comparison with the heat transfer due to the phase transitions, and also that the gap 2h between the skins is relatively narrow: $h/r_0 \ll 1$ (Fig. 1). Then the parameters of the sublimate flowing in the gap can be characterized by a potential Φ, which is introduced as in [2]; to determine this we consider the median surface of the conical slot and bring it into coincidence with the complex plane $Z = r\exp(i\gamma)$, where $\gamma = \varphi\sin\beta$ (Fig. 2). We follow [2] and put

$$\nabla^2\Psi=\frac{3\mu J_m}{2h^3};\quad \Psi=\int_{p_*}^{p}\Phi(p)\,dp;$$

$$\Phi=\frac{p}{RT}+\frac{2-\theta}{\theta}\cdot\frac{3.78\,\mu}{h\sqrt{RT}}-\frac{9}{4}\cdot\frac{\mu^2}{h^2}\cdot\frac{d\ln F(p)}{dp};\tag{3}$$

$$F(p)=\frac{RT_*/L}{1-(RT_*/L)\ln(p/p_*)};\quad \nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial}{\partial r}+\frac{1}{r^2}\cdot\frac{\partial^2}{\partial\varphi^2}.$$

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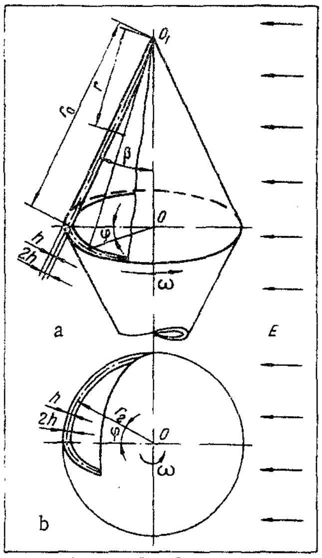

Fig. 1. Calculation of a thermostatic jacket: a) conical; b) spherical.

In (3) we have neglected the additional speed of the sublimate due to the rotation of the ring channel, since the angular velocity of the latter required for reliable operation is only small: $\omega \ll V/(r_0 \sin\beta)$.

The temperature of the outer surface of the skin T_W is related to $T(p) = T_* F(p)$ at the phase-transition surface by

$$T_w - T(p) = qR_w;\ R_w = \frac{\delta_w}{\lambda_w} + \frac{\delta'}{\lambda'}. \tag{4}$$

The use of Ψ enables us to describe the flow in the slot by means of (3), which contains small nonlinearity on the right, which is due to the temperature dependence of the specific heat flux q. For this reason, T_W^4 is best expanded as a Taylor series around some value T_0^4; we put $t(p) = T(p) - T_0$; $t(p) \ll T_0$, and if we restrict consideration to the first two terms, we get from (1) and (4) that

$$q = \frac{A_s E \cos\beta f(\varphi) - \varepsilon\sigma T_0^4 [1 + 4t(p)/T_0]}{1 + 4\varepsilon\sigma T_0^3 R_w}, \tag{5}$$

$$f(\varphi) = \begin{cases} 0 & \text{for } \varphi < \pi/2 \text{ and } \varphi > 3\pi/2, \\ -\cos\varphi & \text{for } \pi/2 < \varphi < 3\pi/2. \end{cases}$$

As $\Delta p/T_0 \cdot dT/dp \ll 1$ (Δp is the pressure difference in the gap), we have

$$t(p) = \frac{(dT/dp)_0}{(d\Psi/dp)_0} (\Psi - \Psi_0) = \frac{RT_0^2}{Lp_0 \Phi(p_0)} (\Psi - \Psi_0). \tag{6}$$

Since the second term on the right in (2) is negligible, as we shall see subsequently, if L and, also, ω are small (provided the values are sufffficient for efficient operation), we use (2),(5), and (6) to express (3) as

$$\nabla^2 \Psi = D + \varepsilon HN\Psi - MHA_s E \cos\beta f(\varphi). \tag{7}$$

Here

$$H = (1 + 4\varepsilon\sigma T_0^3 R_w)^{-1};\ M = \frac{3\mu}{2h^3 L};$$

$$N = \frac{6\sigma R T_0^5 \mu}{h^3 L^2 p_0 \Phi(p_0)};\ D = M\varepsilon H\sigma T_0^4 + 2N\varepsilon H\Psi_0.$$

We can consider the flow of sublimate as symmetrical with respect to the meridional plane passing through the generator $\varphi = 0$, i.e., Ψ should satisfy the boundary conditions

$$\partial\Psi/\partial\gamma = 0 \text{ for } \gamma = 0 \text{ and for } \gamma = \pi \sin\beta. \tag{8}$$

As the flows are symmetrical with respect to the plane $r = r_0$, we have

$$\partial\Psi/\partial r = 0 \text{ for } r = r_0. \tag{9}$$

To solve the boundary-value problem of (7)-(9) we perform a conformal mapping of the sector $0 < r < r_0$; $|\gamma| < \pi \sin\beta$ on a circle of radius $r_{10} = r_0^{1/\sin\beta}$, by means of the analytical function $Z_1 = Z^{1/\sin\beta}$; $Z_1 = r_1 \exp(i\gamma_1)$; $r_1 = r^{1/\sin\beta}$; $\gamma_1 = \gamma/\sin\beta = \varphi$.

Then $Z_1 = r_1 \exp(i\varphi)$; since

$$\frac{dZ_1}{dZ} = \frac{r_1^{1-\sin\beta}}{\sin\beta},$$

(7) becomes

$$\nabla_1^2 \Psi = r_1^{-2(1-\sin\beta)} \sin^2\beta\, [D + \varepsilon HN\Psi - MHA_s E \cos\beta f(\varphi)], \tag{10}$$

$$\nabla_1^2 = \frac{\partial^2}{\partial r_1^2} + \frac{1}{r_1} \cdot \frac{\partial}{\partial r_1} + \frac{1}{r_1^2} \cdot \frac{\partial^2}{\partial\varphi^2}.$$

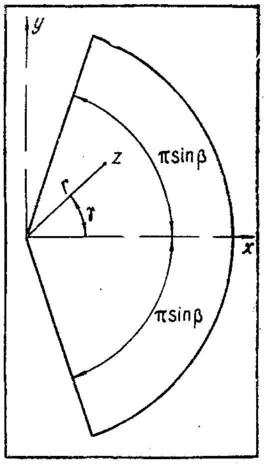

Fig. 2. Coincidence of the median surface of a slot channel with the Z plane.

It is shown below that $Nr_0^2 \ll 1$ for modes of operation of practical interest, so Ψ is sought as an expansion ($N_0 = Nr_0^2$)

$$\Psi = \Psi^{(0)} + N_0\Psi^{(1)} + N_0^2\Psi^{(2)} + \ldots \tag{11}$$

The function $\Psi^{(0)}$ should satisfy (9) and Poisson's equation

$$\nabla_1^2\Psi^{(0)} = r_1^{-2(1-\sin\beta)}\sin^2\beta\,[D - MHA_sE\cos\beta f(\varphi)]. \tag{12}$$

The potential $\Psi^{(0)}$ will be sought in the form $\Psi^{(0)} = \Psi_0^{(0)} + \Psi_1^{(0)}$, where $\Psi_0^{(0)}$ is a particular solution to (12) and $\Psi_1^{(0)}$ is a function harmonic in the Z_1 plane that satisfies the following condition at $r_1 = r_{10}$:

$$\partial\Psi_1^{(0)}/\partial r_1 = -\partial\Psi_0^{(0)}/\partial r_1. \tag{13}$$

To determine $\Psi_0^{(0)}$ we expand $f(\varphi)$ as a Fourier series:

$$f(\varphi) = \sum_{n=0}^{\infty} a_n\cos(n\varphi);\quad a_0 = \frac{1}{\pi};\quad a_1 = -\frac{1}{2};\ a_{2k} = \frac{2(-1)^{k+1}}{(4k^2-1)\pi},$$

$$a_{2k+1} = 0;\quad k = 1,\ 2,\ \ldots\ .$$

Then $\Psi_0^{(0)}$ may be put as the series

$$\Psi_0^{(0)} = \sin^2\beta\sum_{n=1}^{\infty} b_n\Psi_{0n}^{(0)},$$

$$b_n = -MHA_sEa_n\cos\beta;\quad \nabla_1^2\Psi_{0n}^{(0)} = r_1^{-2(1-\cos\beta)}\cos(n\varphi).$$

Since we have that

$$\nabla_1^2 = 4\ \frac{d}{dZ_1}\cdot\frac{d}{d\bar{Z}_1};\quad \cos(n\varphi) = \frac{Z_1^n + \bar{Z}_1^n}{2r_1^n};\quad r_1 = (Z_1\bar{Z}_1)^{1/2},$$

the latter equation can be put in complex form:

$$\frac{d}{dZ_1}\cdot\frac{d\Psi_{0n}^{(0)}}{d\bar{Z}_1} = \frac{1}{8}\left(\frac{Z_1^{n/2-1+\sin\beta}}{\bar{Z}_1^{n/2+1-\sin\beta}} + \frac{\bar{Z}_1^{n/2-1+\sin\beta}}{Z_1^{n/2+1-\sin\beta}}\right).$$

We integrate the right side successively with respect to Z_1 and $\bar{Z}_1$ to obtain

$$\Psi_{0n}^{(0)} = -\frac{1}{2}\cdot\frac{1}{n^2-4\sin^2\beta}\cdot\frac{Z_1^n + \bar{Z}_1^n}{(Z_1\bar{Z}_1)^{n/2-\sin\beta}} = -\frac{r_1^{2\sin\beta}\cos(n\varphi)}{n^2-4\sin^2\beta}. \tag{14}$$

For $\beta = \pi/6$ at $n = 1$ a solution in the form of (14) would be meaningless, and in that case

$$\Psi_{01}^{(0)} = \frac{1}{8}\ln\left(\frac{\bar{Z}^z}{Z^{\bar{z}}}\right) = \frac{r_1}{4}(\ln r_1\cos\varphi + \varphi\sin\varphi).$$

If $\beta \neq \pi/6$, then $\Psi_1^{(0)}$ is sought in the same form as $\Psi_0^{(0)}$, i.e.,

$$\Psi_1^{(0)} = \sin^2\beta\sum_{n=1}^{\infty} b_n\Psi_{1n}^{(0)}.$$

Here $\Psi_{1n}^{(0)}$ are harmonic functions that satisfy the following condition at $r_1 = r_{10}$:

$$\frac{\partial\Psi_{1n}^{(0)}}{\partial r_1} = -\frac{\partial\Psi_{0n}^{(0)}}{\partial r_1} = \frac{2\sin\beta r_{10}^{2\sin\beta-1}\cos(n\varphi)}{n^2-4\sin^2\beta}.$$

Then

$$\Psi_{1n}^{(0)} = \frac{2\sin\beta r_{10}^{2\sin\beta}}{n(n^2-4\sin^2\beta)}\left(\frac{r_1}{r_{10}}\right)^n\cos(n\varphi).$$

Consequently, for $\beta \neq \pi/6$ we have

$$\Psi^{(0)} = \sum_{n=1}^{\infty} \frac{b_n r_0^2 \sin^2\beta}{n^2 - 4\sin^2\beta}\left[\frac{2\sin\beta}{n}\left(\frac{r}{r_0}\right)^{n/\sin\beta} - \left(\frac{r}{r_0}\right)^2\right]\cos(n\varphi). \tag{15}$$

For the case $\beta = \pi/6$ we determine $\Psi_1^{(0)}$ from the boundary condition at $r_1 = r_{10}$, which involves a Fourier expansion of $\varphi\sin\varphi$; in other respects, the procedure for finding $\Psi_1^{(0)}$ does not differ from the general case. We then have the final result for $\beta = \pi/6$:

$$\Psi^{(0)} = \frac{r_0^2}{4}\left\{\frac{b_1}{4}\left(\frac{r}{r_0}\right)^2\left[\left(2\ln\frac{r}{r_0} - \frac{1}{2}\right)\cos\varphi + \varphi\sin\varphi\right]\right.$$

$$\left. + \sum_{n=2}^{\infty}\frac{b_n}{n^2-1}\left[\frac{1}{n}\left(1 + \frac{1}{2}\frac{b_1}{b_n}\right)\left(\frac{r}{r_0}\right)^{2n} - \left(\frac{r}{r_0}\right)^2\right]\cos(n\varphi)\right\}.$$

The determination of all the subsequent $\Psi^{(k)}$ resembles that of $\Psi^{(0)}$ in amounting to solution of a Poisson equation whose right side contains the functions derived in the preceding stages; the boundary conditions for $\partial\Psi^{(k)}/\partial r_1$ remain homogeneous as before, so all the terms in the asymptotic expansion of (11) can be determined by the above scheme. For instance, if we substitute (15) onto the right in (10) we get for $\beta \neq \pi/6$

$$\frac{d}{dZ_1}\cdot\frac{d\Psi^{(1)}}{d\bar{Z}_1} = \frac{\varepsilon H\sin^4\beta}{4}\sum_{n=1}^{\infty}\frac{b_n}{n^2 - 4\sin^2\beta}\left[\alpha_n'(Z_1, \bar{Z}_1) + \overline{\alpha_n'(Z_1, \bar{Z}_1)} - \alpha_n''(Z_1, \bar{Z}_1) - \overline{\alpha_n''(Z_1, \bar{Z}_1)}\right],$$

$$\alpha_n''(Z_1, \bar{Z}_1) = \frac{1}{2r_{10}^{2\sin\beta}} Z_1^{n/2+2\sin\beta-1}\bar{Z}_1^{2\sin\beta-1-n/2},$$

$$\alpha_n'(Z_1, \bar{Z}_1) = \frac{\sin\beta}{nr_{10}^n} Z_1^{n-1+\sin\beta}\bar{Z}_1^{-(1-\sin\beta)}.$$

The solution at $r_1 = r_{10}$ to the condition $\partial\Psi^{(1)}/\partial r_1 = 0$ for the latter equation takes the form

$$\Psi^{(1)} = \frac{\varepsilon H\sin^4\beta r_0^2}{2}\sum_{n=1}^{\infty}\frac{b_n}{n^2 - 4\sin\beta}\left\{\frac{1}{n(n+\sin\beta)}\left(\frac{r}{r_0}\right)^{n/\sin\beta+2} + \frac{2}{n^2 - 16\sin^2\beta}\left(\frac{r}{r_0}\right)^4\right.$$

$$\left. - \left[\frac{n+2\sin\beta}{n^2(n+\sin\beta)} + \frac{8\sin\beta}{n(n^2 - 16\sin^2\beta)}\right]\left(\frac{r}{r_0}\right)^{n/\sin\beta}\right\}\cos(n\varphi). \tag{16}$$

We determine the unknown temperature of the sublimate at $r = r_0$, $\varphi = 0$ [$T(r_0; 0) = T_0$; $\Psi(r_0, 0) = \Psi_0$] by equating to zero the integral over the surface of the cone ($0 < \varphi < 2\pi$, $0 < r < r_0$) on the right in (10), which is proportional to the specific heat flux to the outer skin; we use (15) to get the following transcendental equation accurate up to $N_0 = r_0^2 N$ for this case [$N_0 = N_0(T_0)$]:

$$T_0^4 = \frac{A_s E\cos\beta}{\pi\varepsilon\sigma}\left\{1 - N_0\varepsilon H\sin^2\beta\left[\frac{\pi}{2(1+2\sin\beta)} - \frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(4k^2-1)(k+\sin\beta)k}\right]\right\}. \tag{17}$$

As in the derivation of (7), we neglect the second term on the right in (2) to get the equation for the stationary desublimate distribution:

$$\frac{d\delta'}{d\varphi} = -\frac{q}{\omega L\rho'}.$$

Consequently,

$$\delta' = \delta_0' + \frac{HA_s E\cos\beta}{\omega L\rho'}\left[\frac{\varphi}{\pi} - f_1(\varphi)\right] + 0(N_0), \tag{18}$$

$$f_1(\varphi) = \begin{cases} 0 & \text{for } 0 < \varphi < \pi/2 \text{ and } 3\pi/2 < \varphi < 2\pi, \\ 1 - \sin\varphi & \text{for } \pi/2 < \varphi < 3\pi/2. \end{cases}$$

By virtue of (6), the maximum temperature difference at the sublimation surface is

$$\Delta T = T(r_0, \pi) - T(r_0, 0) = -\frac{3\mu RT_0^2 r_0^2 \sin^2\beta \cos\beta HA_sE}{L^2 p_0 \Phi(p_0) h^3}$$

$$\times \sum_{n=1}^{\infty} a_n \left\{ \frac{1}{n(n+2\sin\beta)} - \frac{N_0 \varepsilon H}{n^2 - 4\sin^2\beta} \left[\frac{1}{n(n+4\sin\beta)} - \frac{\sin\beta}{n^2(n+\sin\beta)} \right] \right\}. \quad (19)$$

As an illustration we consider a conical thermostat rotating around an axis perpendicular to the solar radiation flux ($E = 1392$ W/m^2); with $A_s = 0.15$, $\varepsilon = 0.18$, $\beta = \pi/4$, and $R_w = 0$, $T_0 = [1 + 0(N_0)]\cdot 260$°K, and if the working body is water ($L = 3.06\cdot 10^6$ J/kg, $R = 461.36$ m^2/sec^2 °K, $\mu = 0.81\cdot 10^{-5}$ N·sec/m^2), then $p_0 \approx 182$ N/m^2, and for $r_0 = 2$ m, $2h = 10$ and 20 mm the dimensionless parameter $N_0 = 3.78\cdot 10^{-3}$ and $4.73\cdot 10^{-4}$, while the error in the thermostatic control can be estimated in terms of the maximum temperature differences $\Delta T \approx 0.123$°K and 0.0154°K, respectively.

If we neglect quantities of order N_0, we find that for $r = r_0$ the maximum and minimum in the thickness δ' of the layer of desublimate correspond to the following angles: $\varphi = \arccos(-1/\pi)$ and $\varphi = 2\pi - \arccos(-1/\pi)$:

$$\delta'_{max} = \delta'_0 + 0.545\,\frac{HA_sE\cos\beta}{\omega L\rho'}; \quad \delta'_{min} = \delta'_0 - 0.545\,\frac{HA_sE\cos\beta}{\omega L\rho'}.$$

If δ' is not to vary with φ by more than $\delta'_2 - \delta'_1$, the thermostat must rotate with the angular velocity

$$\omega \geqslant \omega_0 \approx 1.09\,\frac{HA_sE\cos\beta}{L\rho'(\delta'_2 - \delta'_1)}. \quad (20)$$

The minimum necessary angular velocities for $R_w = 0$ ($H = 1$), $\beta = \pi/4$, and $\delta'_2 - \delta'_1 = 0.05, 0.10$, and 1.00 mm are $\omega_0 = 9.95\cdot 10^{-4}$, $5.25\cdot 10^{-4}$, and $0.525\cdot 10^{-4}$ sec^{-1}, i.e., the thermostat must perform a rotation in time intervals not exceeding 1.655, 3.31, and 33.1 h, respectively.

We use (3), (5), (6), and (20) to estimate the ratio of the second term on the right in (2), which characterizes the thermal inertia of the layer of desublimate (and skins), to the first term, which characterizes the heat input:

$$\xi \sim \frac{\omega(c'\rho'\delta'_0 + c_w\rho_w\delta_w)\,dT/d\varphi}{q} \sim \frac{c'\delta'_0 RT_0^2 \mu r_0^2 \sin^2\beta HA_sE}{L^3 p_0 \Phi(p_0)(\delta'_2 - \delta'_1) h^3}\left(1 + \frac{c_w\rho_w\delta_w}{c'\rho'\delta'_0}\right)\cos\beta.$$

For water with $\delta'_2 - \delta'_1 = 0(\delta'_0)$, $A_3 = 0.15$, $\beta = \pi/4$, $\varepsilon = 0.18$ ($T_0 = 260$°K), $R_w = 0(H = 1)$, $c_w\rho_w\delta_w/c'\rho'\delta'_0 \to 0$, $E = 1392$ W/m^2, $r_0 = 2$m, and $2h = 10$ and 20 mm, and we have, respectively, $\xi \sim 0.0034$ and 0.00043.

The above case corresponds to constant height of the slot between conical skins, and it represents a monotonic increase in the temperature differences $\Delta T(r) = T(r, \pi) - T(r, 0)$ as one recedes from the vertex of the cone (as r increases); the thermodynamic parameters of the sublimate are uniquely related to Ψ (apart from an additive constant), and they are dependent not only on φ but also on r.

The dependence of Ψ and ΔT on r can be eliminated by appropriate shaping of the slot; if $Kn < 0.01$, we can neglect the effects of slip, and for $\Delta h \neq 0$ we have (3) for Ψ in the following form [2]:

$$\nabla(h^3\nabla\Psi) = -\frac{3\mu J_m}{2}; \quad \Phi(p) = \frac{p}{RT}.$$

We seek the gap profile in the form $h = h_0(r_1/r_{10})^n$; then in place of (10) we have in the Z_1 plane that

$$\frac{\partial^2\Psi}{\partial\varphi^2} + r_1^2\frac{\partial^2\Psi}{\partial r_1^2} + (3n+1)\,r_1\frac{\partial\Psi}{\partial r_1} = (r_{10}^n/h_0)^3\, r_1^{2\sin\beta - 3n}\,[D_1 + \varepsilon HN_1\Psi - M_1HA_sE\cos\beta f(\varphi)]. \quad (21)$$

Here $D_1 = h^3D$, $N_1 = h^3N$, $M_1 = h^3M$, $\partial D_1/\partial h = 0$, $\partial N_1/\partial h = 0$, $\partial M_1/\partial h = 0$; if the solution to (21) that satisfies (9) is not be to dependent on r, we should put $n = (2\sin\beta)/3$, i.e., $h = h_0(r/r_0)^{2/3}$.

Similarly, we can consider the phase transition in the relatively narrow gap 2h between two spherical skins whose radii are $r_2 - h$ and $r_2 + h$ ($h/r_2 \ll 1$); the conditions for external heat transfer are assumed to be the same as in the conical skins, i.e.,

$$q = \begin{cases} -\varepsilon\sigma T_w^4 & \text{for } 0 < \varphi < \pi/2, \\ -\varepsilon\sigma T_w^4 - A_sE\cos\varphi & \text{for } \pi/2 < \varphi < \pi. \end{cases} \quad (22)$$

The potential Ψ (introduced above) here should satisfy the equation of conservation of matter (symbols as before):

$$K\Psi = -\frac{3\mu r_2^2 J_m}{2h^3}; \quad K = \frac{1}{\sin\varphi} \cdot \frac{d}{d\varphi}\left(\sin\varphi \frac{d}{d\varphi}\right). \tag{23}$$

Equations (7) and (8) will now give

$$K\Psi = r_2^2 [D + \varepsilon HN\Psi - MHA_s Ef(\varphi)], \tag{24}$$

$$\partial\Psi/\partial\varphi = 0 \quad \text{for} \quad \varphi = 0 \text{ and for } \varphi = \pi. \tag{25}$$

The potential Ψ is sought in the form of (11); $\Psi^{(0)}$ is a solution to

$$K\Psi^{(0)} = r_2^2 [D - MHA_s Ef(\varphi)]; \ \partial\Psi/\partial\varphi = 0 \quad \text{for} \quad \varphi = 0, \ \pi. \tag{26}$$

Then

$$\Psi^{(0)}(\varphi) = \Psi^{(0)}(0) - \frac{3\mu r_2^2 HA_s E}{4h^3 L} f_2(\varphi),$$
$$f_2(\varphi) = \begin{cases} \ln\cos(\varphi/2) & \text{for} \quad 0 < \varphi < \pi/2, \\ \ln\sin(\varphi/2) + \cos\varphi & \text{for} \quad \pi/2 < \varphi < \pi. \end{cases} \tag{27}$$

The derivation of all the subsequent $\Psi^{(k)}$ amounts to solving analogous problems $K\Psi^{(k)} = \varepsilon H\Psi^{(k-1)}$; $\partial\Psi^{(k)}/\partial\varphi = 0$ for $\varphi = 0; \pi$.

For instance,

$$\Psi^{(1)}(\varphi) = \Psi^{(1)}(0) - \frac{3\mu r_2^2 \varepsilon HA_s E}{4h^3 L} F(\varphi),$$
$$F(\varphi) = \begin{cases} 2\mathrm{I}(\cos\varphi/2) - (\ln 2 - 1/2)\ln(\cos\varphi/2) & \left(0 < \varphi < \frac{\pi}{2}\right), \\ 2\mathrm{I}(\sin\varphi/2) - (\ln 2 - 1/2)\ln(\sin\varphi/2) + \frac{\cos\varphi}{2} & \left(\frac{\pi}{2} < \varphi < \pi\right), \end{cases} \tag{28}$$
$$\mathrm{I}(\alpha) = \int_\alpha^1 \frac{x\ln x}{1 - x^2} dx.$$

The maximum temperature difference at the sublimation surface is

$$\Delta T = \frac{3\mu r_2^2 RT_0^2 HA_s E}{4L^2 p_0 \Phi(p_0) h^3}\left[1 - \frac{3\sigma\mu r_2^2 RT_0^5 \varepsilon H}{h^3 Lp_0 \Phi(p_0)} + 0(N_0^2)\right]. \tag{29}$$

The thermodynamic parameters $[p_0, \Phi(p_0)]$ can be determined from the Clausius–Clapeyron equation, as (3) shows, if we use

$$T_0 = \sqrt[4]{\frac{A_s E}{4\varepsilon\sigma}}\left[1 - \frac{N_0 \varepsilon H}{2}\left(\ln 2 - \frac{3}{2}\right) + 0(N_0^2)\right]. \tag{30}$$

One cannot maintain steady thermostatic conditions by virtue of phase transitions indefinitely long in a spherical gap by rotating a sphere around a single axis, on account of change in the conditions for external heat transfer along the generators; such a thermostat can work in pulse mode, i.e., for a finite time t_1 corresponding to elimination of the desublimate layer at the point $\varphi = \pi$ for $\omega = 0$ or along a great circle on the spherical surface in the plane of rotation if ω is large enough. If we neglect quantities of the order of N_0, the time t_1 for the above cases is defined as follows:

$$t_1' = \frac{4}{3} \cdot \frac{\rho L\delta_*}{HA_s E} \quad \text{for} \quad \omega = 0; \ t_1'' = \frac{1}{1/\pi - 1/4} \times \frac{\rho L\delta_*}{HA_s E} \quad \text{for} \quad \omega \gg \frac{2\pi}{t_1'}.$$

Here δ_* is the initial thickness of the desublimate layer. The thermostatic control may be considered as continuous if within time intervals $t_2 < t_1''$ there is a change in the orientation of the axis of rotation with respect to the plane of rotation for $\omega \gg 2\pi/t_1'$.

NOTATION

r, φ	are spherical coordinates of points on the median slot channel surface;
2β	is the angle at cone apex;
$2h$	is the slot channel height;
$Z = re^{i\gamma}$ and $Z_1 = r_1e^{i\varphi}$	are the points on corresponding complex planes;
T_w, $T(p)$, T_m	are the temperatures of external thermostat surface, saturation temperature at pressure p, and temperature averaged over the desublimate layer plus over outer shell, respectively;
q	is the heat flux density at outer shell, W/m^2;
E	is the shortwave radiation flux density, W/m^2;
A_S, ε	are the shortwave radiation absorption factor and emissivity of external surface;
σ	is Stefan's constant;
c', ρ', δ', (c_w, ρ_w, δ_w)	are the specific heat, density, and thickness of desublimate layer (outer shell), respectively;
L	is the latent heat of sublimation; J/kg;
p	is the pressure, N/m^2;
p_*, T_*	are the sublimate parameters at triple point;
R	is the gas constant of sublimate, $m^2/sec^2 \cdot °K$ (J/kg · °K);
λ_w, λ'	are the thermal conductivities of outer shell and desublimate layer, respectively, W/m · °K;
$\bar{a}$	is the complex conjugate of a.

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